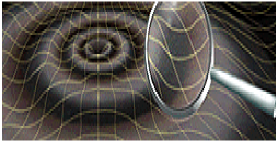

Multi-detector GWave Coherent search Veto

Archana Pai

Indian Institute of Science Education and Research, Thiruvananthapuram

Jamia Milia, 09 February 2011

What are Gravitational Waves?

Einstein's General Relativity (GR) – Bulk motion of the physical systems radiates away asymmetries via GWaves

- GWaves $\equiv$ Ripples of the space-time fabric
- Travel with speed of light
- *Strain Amplitude* $h = \frac{2G}{rc^4}\frac{d^2Q}{dt^2}$
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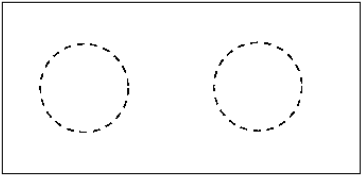

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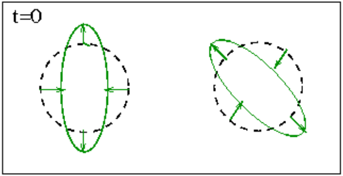

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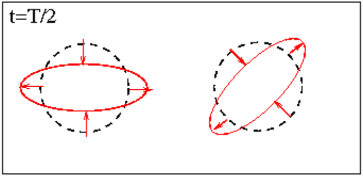

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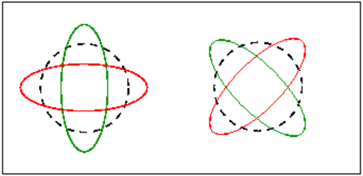
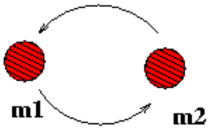


GWaves from Compact Binaries $\quad 1pc = 3.26 light\ yrs$



Typical equal mass Binary system
Total Mass: $m = 1.4M_\odot$,
Orbital radius: $R = 10^6 km$
Orbital period: $7.75hrs$,
Distance: $r = 5kpc = 1.5 \times 10^{17} m$
$(KE)_{nonsp} \sim MR^2\omega^2 = 10^{39} kg\ m^2/s^2$

$$h \sim \frac{G(KE)_{nonsp}}{rc^4} \sim 10^{-21}$$

GWaves from Compact Binaries $\quad 1pc = 3.26 light\ yrs$

Masses, Sky Location, Distance, Polarisation, TOA, POA

Waveform:

$h_+(t) = A_+(t) \cos \Phi(t)$

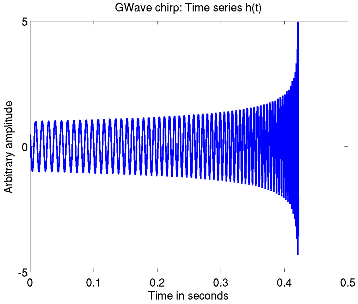
$h_\times(t) = A_\times(t) \sin \Phi(t)$

Freq.: $f \propto \mathcal{M}^{-5/8}(t_{coal} - t)^{-3/8}$

Amp: $A_{+,\times}(t; \epsilon, r, \mathcal{M}) \propto r^{-1}$

$\propto \mathcal{M}^{5/3}$

$\propto f^{2/3}$



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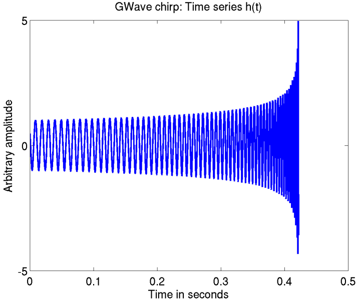

- Chirp duration $\tau_0 \propto \mathcal{M}^{-5/3}$, Smaller the masses $\rightarrow$ Longer the chirp $M = 2.8 M_\odot, \tau_0 = 25 sec, f_s = 40 Hz$
- Detector response
  $s(t) = F_+\ h_+(t) + F_\times\ h_\times(t) = \mathcal{A}(t; \theta^\alpha) \cos(\Phi(t) + \chi(\theta^\alpha))$
  Can not separate all parameters using single detector
- Power spectrum of chirp $|s(f)|^2 \propto f^{-7/3}$

Matched Filtering : Detecting Compact Binaries

GWave detection: Weak signal embedded in the noisy data
Known spectral shape signal == Matched Filtering is optimal

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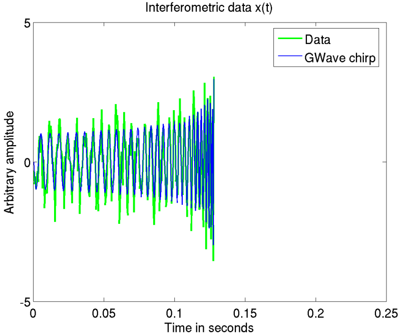

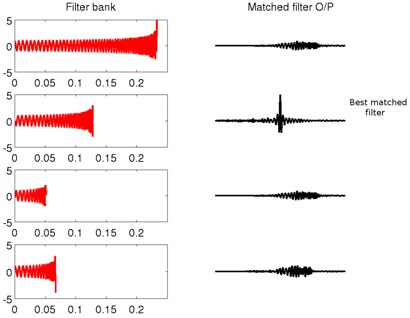

GWave detection is a statistical problem

GWave Detection:

- Known shape – Matched filtering/Maximum Likelihood approach
- Filter the data through the template bank spanning the parameter space
- Pick up that template which maximizes the LR; LR_{max}
- Estimate the false alarm rate from the instrument, obtain the threshold L_0
- Check if $LR_{max} >< L_0$

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GWave Vetos:

Veto events of noise origin which mimic like GWave transients

- Check correlations with the oscilliary channels
- χ^2 veto — Allen 1999
- r^2 veto — Shawhan and Ochsner 2004

χ^2 Veto – Allen PRD 1999

To separate non-Gaussian noise transients from the binary transients

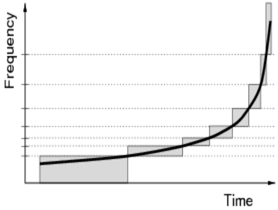
- Idea: Check consistency of event power with the binary inspiral
- Divide frequency band in p sub-templates

$$C_l(t) = 4 \int_{f_k}^{f_{k+1}} \tilde{q}(f)\tilde{x*}(f)/N(f)e^{2\pi ift}df$$

- Note that $< C_l(t) >= C(t)/p$
  Chirp frequency increases monotonically with time
  Construct $\chi^2 = p\sum_l |C_l(t) - C(t)/p|^2 = p\sum_l |\Delta C_l|^2$
- Noise is Gaussian: ΔC_l is gaussian RV and χ^2 obeys chi2-distribution with $2p - 2$ DOF $\Rightarrow \chi^2$ is small.
- Non-Gaussian noise: Makes χ^2 large. Threshold on χ^2.

r^2 Veto – Shawhan and Ochsner 2004

Feature: χ^2 veto for large signal amplitude inspirals



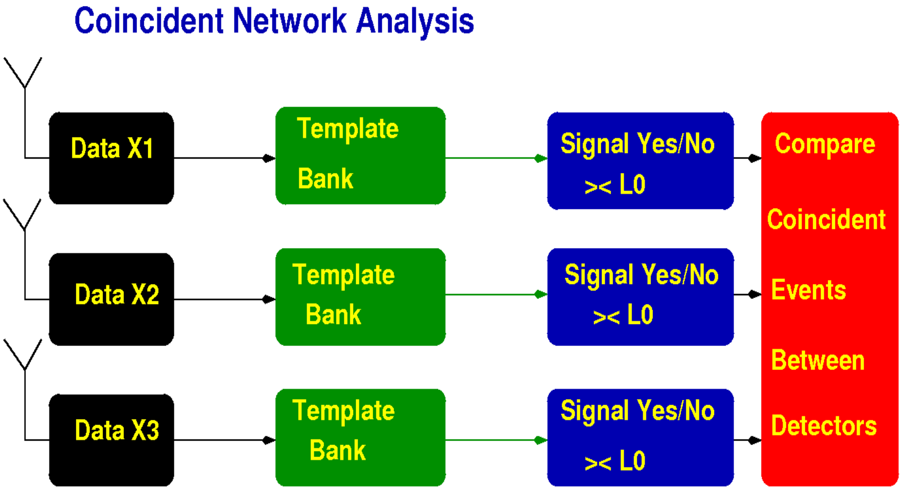
Property of χ^2 statistics:
1/ Outside the chain of boxes, no other region in time-frequency plane affects the χ^2
2/ χ^2 is very sensitive to small mismatch

For large signal amplitude inspirals

- Drawback: Might veto out the actual inspiral signal due to small mismatch
- Idea: Introduce SNR dependent χ^2 threshold $\rightarrow$ r^2 statistic

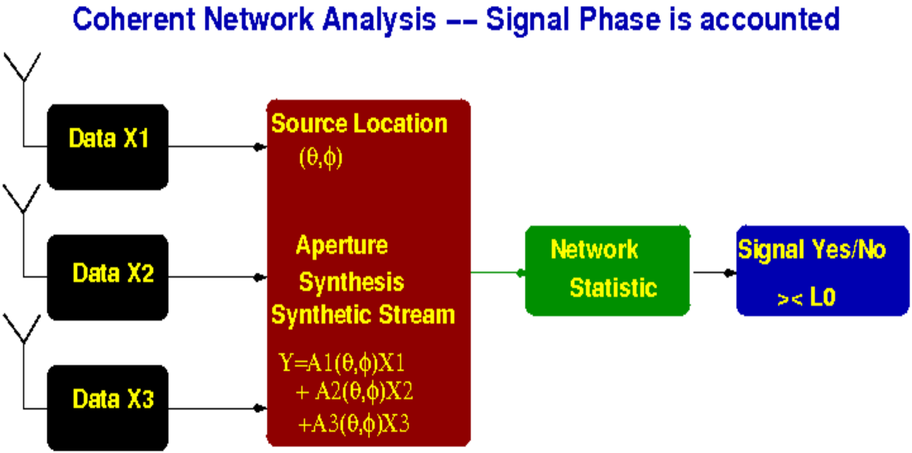
$$\chi^2 < 40 + 0.15\rho^2_{max}$$

Network Schemes



Signal phase is not accounted

Network Schemes



Multi-detector Coherent Formalism for

1. binary Chirps ; [AP, Bose, Dhurandhar PRD 2001]
2. unmodeled chirps – Aperture Synthesis via Synthetic streams [AP, Chassande-Mottin, Rabaste PRD 2008]

Aperture Synthesis : Synthetic Streams and Null Streams

AP,Chassande-Mottin and Rabaste PRD 2008

Maximize Network Likelihood Ratio:

$\Lambda = -\left\|\mathbf{x} - \Pi\mathbb{P}\right\|^2 + \left\|\mathbf{x}\right\|^2 \quad \mathbf{s} = \Pi\mathbb{P} \quad \mathbf{X} = \left[\begin{array}{ccc} \mathbf{x}_1 & \ldots & \mathbf{x}_d \end{array}\right]_{N\times d}$

Solve Linear LSQ:- Pseudo-inverse of Π *i.e.* $\hat{\mathbb{P}} = V_\Pi \Sigma_\Pi^{-1} U_\Pi^H \mathbf{x}$

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$$U_\mathbb{D}^H \underbrace{\mathbf{X}^T U_\varnothing}$$

Project data on to $U_\varnothing$ first and then combine with weights

[AP, Dhurandhar, Bose, PRD 2001]

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(a) Project data on to $U_{\mathbb{D}} \rightarrow$ Synthetic streams
(b) Matched filtering of synthetic streams

[AP, Chassande-Mottin, Rabaste, PRD 2008]

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[AP, Chassande-Mottin, Rabaste, PRD 2008]

For a D detector network
1/ 2 synthetic streams: $\mathbf{Y_1} = \mathbf{Xd_1}$ and $\mathbf{Y_2} = \mathbf{Xd_2}$

$$\hat{\Lambda} \propto |\Phi^H \mathbf{Y_1}|^2 + |\Phi^H \mathbf{Y_2}|^2$$

2/ $D - 2$ Null streams. D=3 gives 1 null stream [Wen,Schutz CQG 2005]

Computational Cost

Coherent detection is expensive :
Example: Newtonian chirp with multi-detectors

Signal detector : $\{t_a, \mathcal{M}, \delta, A\}$
Numerical maximisation : $\mathcal{M}$
Matched filtering technique, scan the $\mathcal{M}$ space
Look for the maximum in the filtered output
Templates: $M = 5000$, $m_1 = m_2 = 0.5 M_\odot$, $N = 10^6$
Comp Cost: $\sim 6 * M * N * log_2 N \rightarrow 1.5 GFlops$

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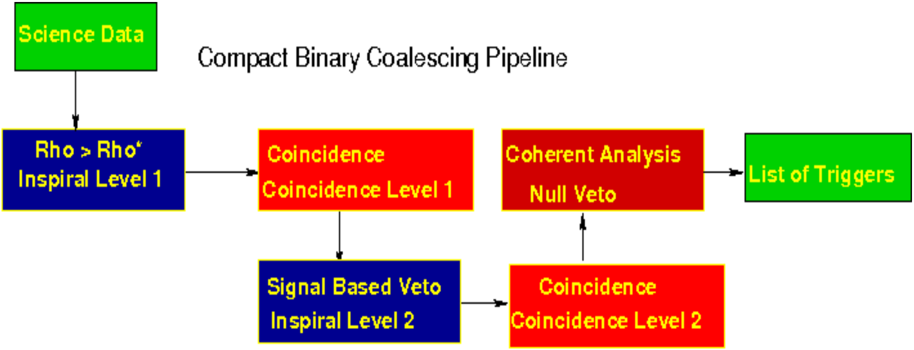
Multi-detectors: $\{t_a, \mathcal{M}, \delta, A, \epsilon, \psi, \theta, \phi\}$ AP, Dhurandhar, Bose 2001
Numerical maximisation : $\mathcal{M}, \theta, \phi$
Matched filtering technique, scan the $\mathcal{M}, \theta, \phi$ space
Look for the maximum in the filtered output
Templates: $\mathcal{M} \sim 7500$, $\Omega \sim 25000 \rightarrow$ *Tens of Tflops*

Proposed work in LSC



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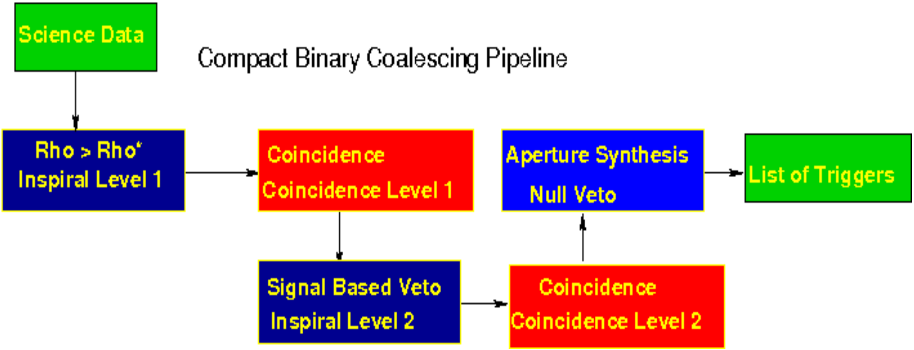


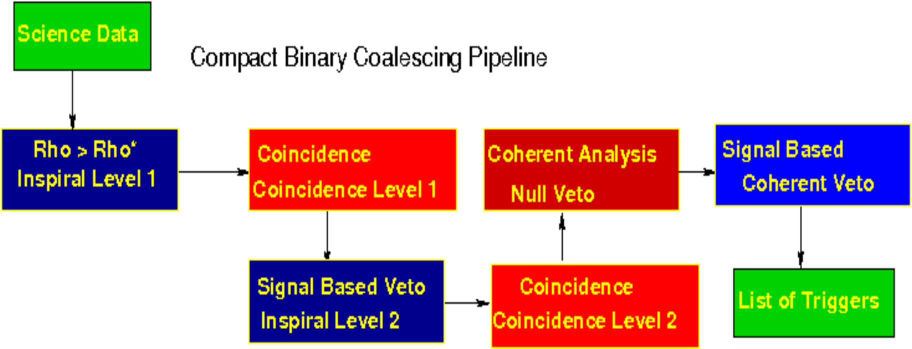
Aim: Low Latency Coherent Search

1. Fold-in aperture synthesis
2. Investigate fast sky search methods

Application:

Targetted Externally trigger GRB search in S6 data

Proposed work in LSC



Aim: Obtain multi-detector χ^2 veto.

Aperture synthesis would give better approach to χ^2 veto

Issues:

1/ Can we fold-in noise features of information to obtain modified χ^2

2/ Criterio for frequency subintervals

Collaborators

People involved in formalism development

- Archana Pai, IISER-TVM
- H. Tagoshi, Osaka University
- Sanjeev Dhurandhar , IUCAA Pune
- Anand S. Sengupta, Delhi University
- N. Kanda, Osaka City University
- H. Takahashi, Yamanashi Eiwa College
- Haris M. K., IISER-TVM, India

IndIGO subgroup involved in implementation in LSC

- Haris M. K., IISER-TVM, India
- Anand S. Sengupta, Delhi University
- Archana Pai, IISER-TVM, India

Japanese subgroup has plans to join LSC